Frekvenční doména

Počítačová grafika

Mgr. Markéta Trnečková, Ph.D.

Palacký University, Olomouc

Spojitá Fourierova transformace

Jednorozměrná

$F(u) = \int_{-\infty}^{\infty} f(x)e^{-i2\pi ux}dx$

Zpětná FT

$f(x) = \int_{-\infty}^{\infty} F(u)e^{+i2\pi ux}dx$

Frekvenční doména

$F(u) = R(u) + iI(u)$

$F(u) = |F(u)|\dot{e}^{i\varphi(u)}$

Modul

$|F(u)| = \sqrt{R^2(u) + I^2(u)}$

Fáze

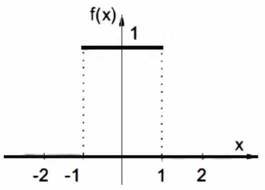
$\varphi(u) = tan^{-1}\left(\frac{I(u)}{R(u)}\right)$

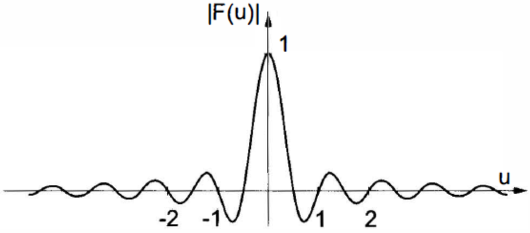
Fourierovo spektrum

$P(u) = |F(u)|^2 = R^2(u) + I^2(u)$

Eulerova formule

$e^{-i2\pi ux} = cos(2\pi ux) - isin(2\pi ux)$





Dvourozměrná Fourierova transformace

Dvourozměrná FT

$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i2\pi(ux+vy)} dxdy$

Zpětná dvourozměrná FT

$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{-i2\pi(ux+vy)} dudv$

Diskrétní Fourierova transformace

FT

$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)e^{-i2\pi\frac{ux}{N}}$

Zpětná FT

$f(x) = \sum_{u=0}^{N-1} F(u)e^{+i2\pi\frac{ux}{N}}$

Dvourozměrná FT

$F(u,v) = \frac{1}{M}\frac{1}{N}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-i2\pi(\frac{ux}{M}+\frac{vy}{N})}$

Zpětná dvourozměrná FT

$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{+i2\pi(\frac{ux}{M}+\frac{vy}{N})}$

Rychlá FT

$F(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) w_N^{ux}$

$w_N = e^{-i2\pi/N} = \cos(-2\pi/N) + i\sin(-2\pi/N)$

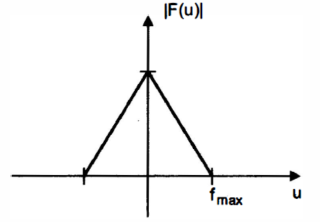
$F(u) = \sum_{x=0}^{N-1} f(x) e^{-i2\pi\frac{ux}{N}} = \sum_{x=0}^{\frac{N}{2}-1} f(2x) e^{-i2\pi u\frac{2x}{N}} + \sum_{x=0}^{\frac{N}{2}-1} f(2x+1) e^{-i2\pi u\frac{2x+1}{N}} =$
$\sum_{x=0}^{\frac{N}{2}-1} f(2x) e^{-i2\pi ux/\frac{N}{2}} + w^u \sum_{x=0}^{\frac{N}{2}-1} f(2x+1) e^{-i2\pi ux/\frac{N}{2}} = F^e(u) + w^u F^o(u)$
$F^e(u + \frac{N}{2}) = F^e(u)$, $F^o(u + \frac{N}{2}) = F^o(u)$
$w^{u+\frac{N}{2}} = -w^u$
$F(u) = F^e(u) + w^u F^o(u)$ pro $0 \leq u \leq N/2$
$F(u + \frac{N}{2}) = F^e(u) - w^u F^o(u)$

RecursiveFFT(f)

N = lenght(f);
if $N = 1$ **then**
  return f;
end
$\omega_n \leftarrow e^{-i2\pi/n}$;
$\omega \leftarrow 1$;
$f^e \leftarrow$ body se sudým indexem;
$f^o \leftarrow$ body s lichým indexem;
$y^e \leftarrow$ **recursiveFFT** (f^e);
$y^o \leftarrow$ **recursiveFFT** (f^o);
for $k \leftarrow 0$ **to** $\frac{n}{2} - 1$ **do**
  $y(k) \leftarrow y^e(k) + \omega y^o(k)$;
  $y(k + \frac{n}{2}) \leftarrow y^e(k) - \omega y^o(k)$;
  $\omega \leftarrow \omega\omega_n$;
end
return y

Shannonův vzorkovací teorém

Signál spojitý v čase je plně určen posloupností vzorků odebíraných ve stejných intervalech $\Delta x \Leftrightarrow f_s = \frac{1}{\Delta x} > 2 \cdot f_{max}$.



Konvoluce

$f(x) * h(x) = \int_{-\infty}^{\infty} f(x-\alpha)h(\alpha)d\alpha$

konvoluční jádro $h(x)$

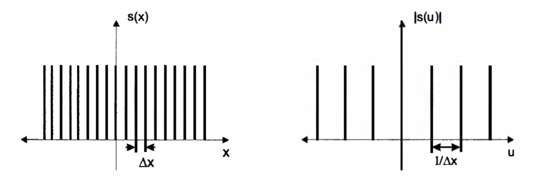
$f(x) * h(x) \Leftrightarrow F(u) \cdot H(u)$
$F(u) * H(u) \Leftrightarrow f(x) \cdot h(x)$

Diskrétní konvoluce

$f(x,y) * h(x,y) = \sum_{a=-k}^{k} \sum_{b=-k}^{k} f(x-a, y-b)h(a,b)$

Vzorkování a konvoluce

Diracův pulz $\delta(x) = 0, \forall x \neq 0$ a $\int_{-\infty}^{\infty} \delta(x)dx = 1$
vzorkovací funkce $s(x) = \sum_{i=-\infty}^{\infty} \delta(x - i\Delta x)$

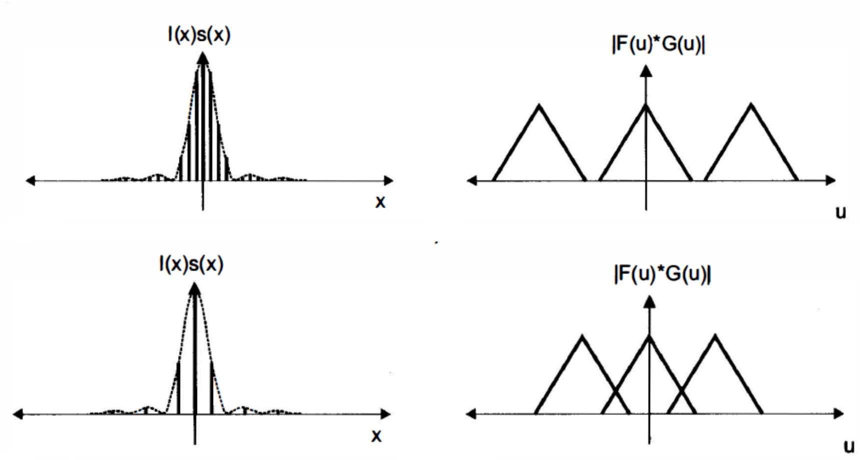


Fourierův obraz
$S(u) = \frac{1}{\Delta x} \sum_{i=-\infty}^{\infty} \delta\left(\frac{u-i}{\Delta x}\right)$

Vzorkování a konvoluce

Vzorkování $f(x)s(x)$

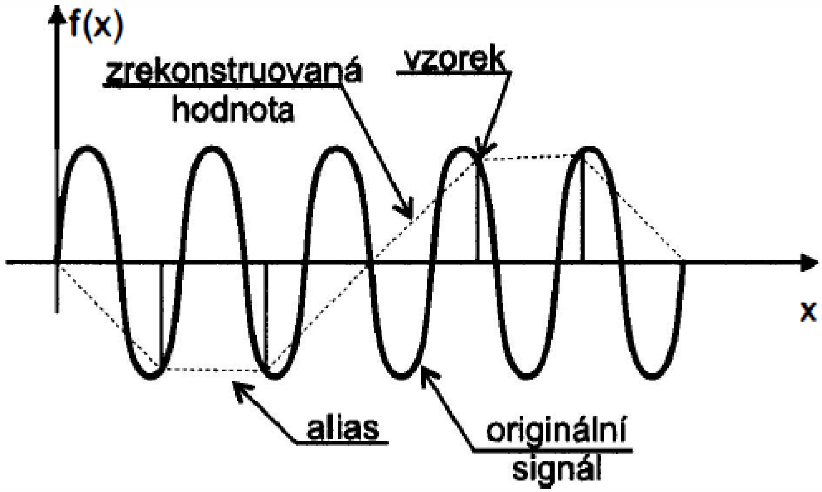
$$I(x) = f(x)s(x) \Leftrightarrow \frac{1}{\Delta x}\sum_{i=-\infty}^{\infty}\delta\left(\frac{u-i}{\Delta x}\right)$$



Matlab

```
F = fft2(f)
S = abs(F)
Fc = fftshift(F)
Fcl = log(1 + abs(F))
F2 = ifftshift(Fc)
f2 = ifft2(F2)
real(f2)
```

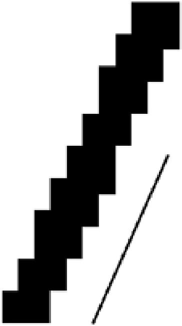
Alias



f(x)
zrekonstruovaná hodnota
vzorek
x
alias
originální signál

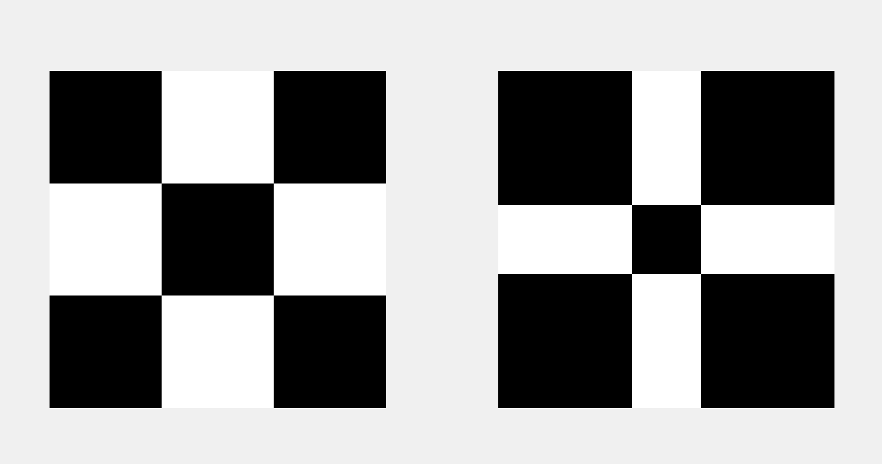
Alias

jaggies

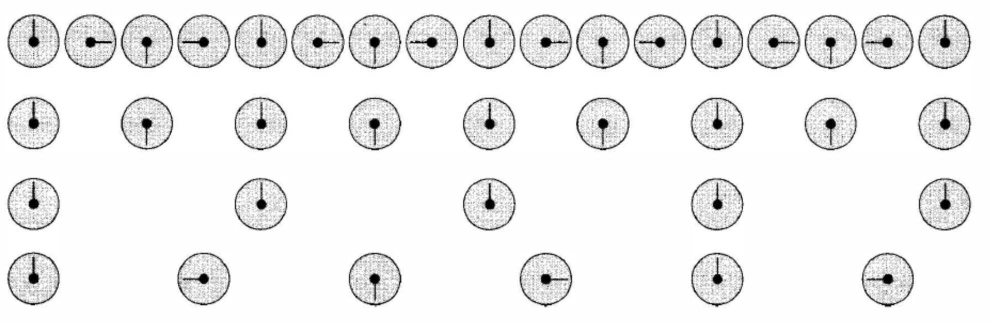


moire

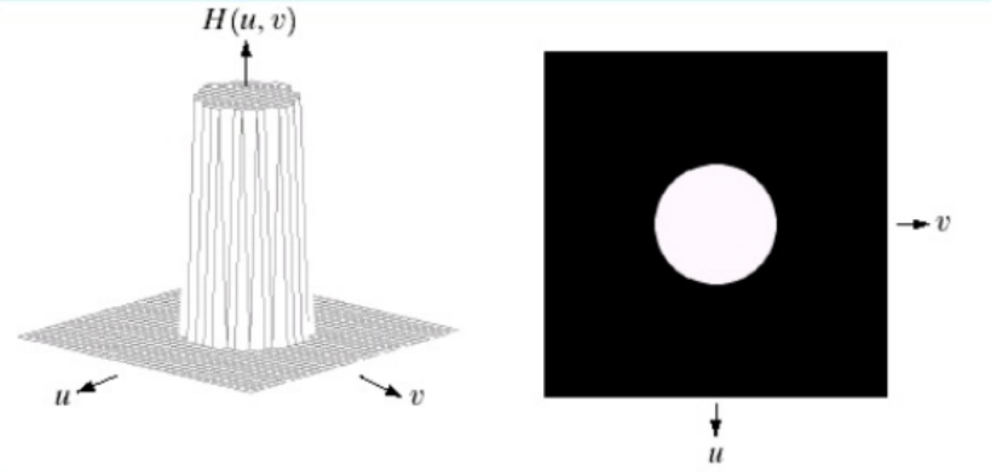
Alias



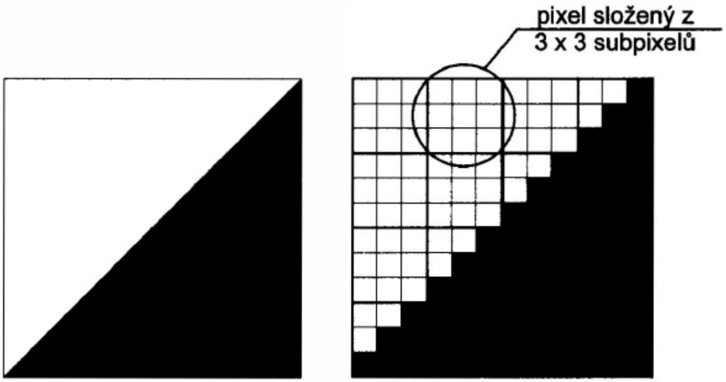
Časový alias



Low pass filter

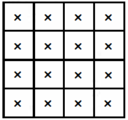


Supersampling

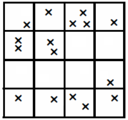


Výběr vzorků

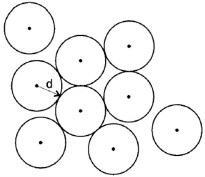
- **Pravidelná mřížka**



- **Rotovaná mřížka**
- **Náhodné vzorkování**

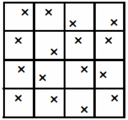


- **Poissonův disk**

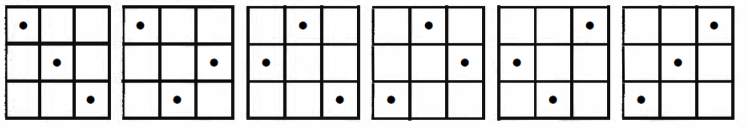


Výběr vzorků

- **Jittering**

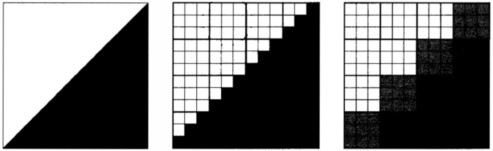


- **N věží**



Výběr vzorků

- **FSAA**



- **HRAA**